

Chapter-2 Polynomials

REVIEW PROBLEMS-2

SHORT ANSWER TYPE PROBLEMS-I

1. Find the zero of the polynomials:

(i) $3x^2 + x - 2$ [CBSE 2010] (ii) $(x - 2)^2 - (x + 2)^2$ [NCERT (EP)]

2. Using Factor Theorem, show that $x^2 + 5x + 6$ is a factor of $x^4 + 5x^2 + 9x + 15x + 18$

[CBSE 2010]

3. Find the value of the polynomial, $f(x) = x^5 - 4x^4 - 7x^3 + 11x - 13$ at $x = 5$

4. Evaluate 105×95 using suitable identity. [CBSE 2012]

5. Determine the value of k such that $(x + 3)$ is a factor of the polynomial

$$f(x) = kx^3 + x^2 - 22x - 21.$$

6. Factorise: $m(m - 1) - n(n - 1)$. [CBSE 2012]

SHORT ANSWER TYPE PROBLEMS-II

7. Factorise the following:

(i) $4a^2 + 12ab + 9b^2 - 8a - 12b$ (ii) $4b^2c^2 - (b^2 + c^2 - a^2)^2$

(iii) $x^8 + x^4 + 1$

(iv) $a^4 b^8 - 2a^2 b^4 c^5 + c^{10}$

(v) $4(x - y)^2 - 12(x - y)(x + y) + 9(x + y)^2$

(vi) $x^4 + 2x^3y - 2xy^3 - y^4$

(vii) $x^2 - 24xy + 135y^2$

(viii) $y^5 - 4y^3 + 3y$

(ix) $x^8 - y^8$ [CBSE 2010]

(x) $x^6 + 26x^3 - 27$

(xi) $1^3 + x^3 + \frac{1}{x^3} - 3$

(xii) $x^3 + 7x^2 + 14x + 8$

8. Factorise each of the following:

(i) $3p^2q^2 + 2p^2q + 9pq^2$ (ii) $3x^2 + 6xy + 5x + 10y$ (iii) $9x^2 - 6xy + y^2$

(iv) $16x^2 + 24xy + 9y^2$

(v) $x^2 - 4y^2$

(vi) $3 - 12(a - b)^2$

[CBSE (Sample Paper) 2010]

(vii) $x^2 + 12x + 20$

(viii) $x^2 - 20x + 96$

(ix) $6x^2 + 17x + 12$

(x) $3x^2 - 10x + 8$

(xi) $6x^2 + 17x + 12$

(xii) $4p^2 - 17p - 21$

(xiii) $2x^2 - x + \frac{1}{8}$

(xiv) $21x^2 - 2x + \frac{1}{21}$

(xv) $2x^2 + 3\sqrt{3}x + 3$

(xvi) $2x^2 + 3\sqrt{5}x + 5$

(xvii) $343 + 27t^3$

(xviii) $8y^3 + 64b^3$

(xix) $27x^3 - 125y^3$ [CBSE 2010]

(xx) $216x^3 - 125y^3$

9. If $x + y + 1 = 0$, prove that $x^3 + y^3 + 1 = 3xy$.

10. If $(2x - 3)$ is a factor of $2x^4 - 3x^2 + 15x - 15k$, find the value of $(3k - \sqrt{5}k)$. [CBSE 2011]

11. Show that $(2x + 3)$ is a factor $f(x) = 2x^3 + 5x^2 - 37x - 60$. Hence, factorise $f(x)$ completely.

12. Find the values of a and b so that the polynomial $f(x) = x^3 - ax^2 - 13x + b$ has $(x - 1)$ and $(x + 3)$ as its factors.

13. Find the values of a and b so that the polynomial $f(x) = 3x^3 + ax^2 - 13x + b$ is exactly divisible by $(x^2 - 2x - 3)$.

14. Find the values of a and b so that the polynomial $f(x) = x^3 + ax^2 + bx - 42$ is exactly divisible by $x^2 - x - 6$.
15. Find h and k so that the polynomial $f(x) = x^3 - hx^2 - 13x + k$ has $(n - 1)$ and $(x + 3)$ factors.
16. Let $f(x) = 6x^4 + 11x^3 + 13x^2 - 3x + 27$ and $g(x) = 3x + 4$.
Find the quotient, $q(x)$ and the remainder, $r(x)$ when $f(x)$ is divided by $g(x)$. Also, check the remainder obtained by using remainder theorem.
17. Verify whether ± 1 , ± 3 are the zero of the polynomial, $f(x) = x^4 + 4x^3 + 2x^2 - 4x - 3$.
18. Without actual division, how will you show that $(x + 4)$ is a factor of the polynomial, $f(x) = x^4 + 2x^3 - 7x^2 + 2x - 8$. Justify your answer and state the theorem you have used.
19. Show, without actual division, that $(x - 2)$ and $(x - 4)$ are the factors of the polynomial, $f(x) = x^4 - 6x^3 + 12x^2 - 24x + 32$.
20. Using suitable identities:
 (i) Evaluate: $(x - 2y)(x + 2y)(x^2 + 4y^2)$
 (ii) Simplify: $(3x - 2y + z)^2 - (3x + 2y - z)^2$
 (iii) Simplify: $(3x + 4)^3 - (3x - 4)^3$
 (iv) Evaluate: $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$. [NCERT (EP)]
 (v) Evaluate: $(x^2 - 1)(x^4 + x^2 + 1)$ [NCERT (EP)]
21. If $a + b + c = 9$ and $ab + bc + ca = 26$, find the value of $a^3 + b^3 + c^3 - 3abc$.
22. If $a + 1 = -2b$, show that $1 + a^3 + 8b^3 = 6ab$.
23. Factorise:
 (i) $64x^6 - y^6$
 (ii) $3x^6 - 81y^6$
 (iii) $18x^3 - 50xy^2$
 (iv) $x^8 - 256$
24. (i) If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$. [NCERT (EP)]
 (ii) If $a^3 + b^3 + c^3 = 3abc$ and $a + b + c = 0$, show that $\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ac} + \frac{(a+b)^2}{3ab} = 1$.

LONG ANSWER TYPE PROBLEMS

25. Simplify: $\frac{(4x^2 - 9y^2)^3 + (9y^2 - 16z^2)^3 + (16z^2 - 4x^2) \times 2}{(2x - 3y)^3 + (3y - 4z)^3 + (4z - 2x)^3}$ [CBSE 2010]
26. Factorise:
 (i) $f(x) = x^3 + 2x^2 - x - 2$ (ii) $f(x) = x^4 + x^3 - 7x^2 - x + 6$ (iii) $x^3 + 13x^2 + 32x + 20$ [CBSE 2012]
27. Factorise: $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$ [CBSE (Sample Paper) 2010]
28. Factorise: $(x^{12} - y^{12})$ [CBSE (Sample Paper) 2010]
29. Prove that:

$$(p+q)^3(p-q)^3(p^2+q^2)^3 + (q+r)^3(q^2+r^2)^3(q-r)^3 + (r+p)^3(r^2+p^2)^3(r-p)^3$$

$$= 3(p^4 - q^4)(q^4 - r^4)(r^4 - p^4)$$
30. The polynomial, $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ when divided by $(x - 1)$ and $(x + 1)$, leaves the remainders 5 and 19 respectively. Find the values of a and b . Hence, find the remainder when $f(x)$ is divided by $(x - 2)$.
31. If the polynomials $(2x^3 + ax^2 + 3x - 5)$ and $(x^3 + x^2 - 2x + a)$ leave the same remainder when divided by $(x - 2)$, find the value of a . Also, find the remainder in each case.
32. Without actual division, prove that $3x^3 - 8x^2 + 3x + 2$ is exactly divisible by $x^2 - 3x + 2$. [CBSE 2012]

ANSWERS

1. (i) $-1, \frac{2}{3}$ (ii) 0
3. -208 4. 9975
7. (i) $(2a + 3b)(2a + 3b - 4)$
 (iii) $(x^2 + 1 + x)(x^2 + 1 - x)(x^4 + 1 - x^2)$
 (v) $(x + 5y)^2$
 (vii) $(x - 15y) \times (x - 9y)$
 (ix) $(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$
 (xi) $\left(1 + x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - x - \frac{1}{x}\right)$
8. (i) $pq(3pq + 2p + 9q)$ (ii) $(x + 2y)(3x + 5)$ (iii) $(3x - y)(3x - y)$
 (iv) $(4x + 3y)(4x + 3y)$ (v) $(x + 2y)(x - 2y)$ (vi) $3[(1 + 2a - 2b)(1 - 2a + 2b)]$
 (vii) $(x + 2)(x + 10)$ (viii) $(x - 8)(x - 12)$ (ix) $(x + 7)(2x - 3)$
 (x) $(x - 2)(3x - 4)$ (xi) $(2x + 3)(3x + 4)$ (xii) $(x + 1)(x + 2)(x + 4)$
 (xiii) $\frac{1}{8}(4x - 1)(4x - 1)$ (xiv) $\frac{1}{21}(21x - 1)(21x - 1)$ (xv) $(x + \sqrt{3})(2x + \sqrt{3})$
 (xvi) $(2x + \sqrt{5})(x + \sqrt{5})$ (xvii) $(7 + 3t)(49 + 9t^2 - 21t)$ (xix) $(3x - 5y)(9x^2 + 25y^2 + 15xy)$
 (xviii) $(2y + 4b)(4y^2 + 16b^2 - 8yb)$
 (xx) $(6x - 5y)(36x^2 + 25y^2 + 30xy)$
10. $k = \frac{1}{40}(207 - 69\sqrt{5})$
12. $a = 3 ; b = 15$
14. $a = 6, b = -13$
16. $q(x) = 2x^2 + x^2 + 3x - 5, r(x) = 47$
20. (i) $x^4 - 16y^4$ (ii) $-24xy + 12xz$ (iii) $216x^2 + 128$ (iv) $\left(\frac{x^3}{8} + 8y^3\right)$ (v) $x^6 - 1$
21. 27.
23. (i) $(2x + y)(2x - y)(4x^2 - 2xy + y^2)(4x^2 + 2xy + y^2)$ (ii) $3(x^2 - 3y^2)(x^4 + 3x^2y^2 + 9y^4)$
 (iii) $2x(3x + 5y)(3x - 5y)$ (iv) $(x^2 + 16)(x^2 + 4)(x + 2)(x - 2)$.
25. $(2x + 3y)(3y + 4z)(4z + 2x)$
26. (i) $(x - 1)(x + 1)(x + 2)$ (ii) $(x - 1)(x + 1)(x - 2)(x + 3)$
 (iii) $(x - 1)(x + 2)(x - 3)$
27. $(a - 4)(a + 2)(a - 5)(a + 3)$.
28. $(x - y)(x + y)(x^2 + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$
30. $a = 5, b = 8; 10$
31. $a = -3$; Remainder in each case = 5